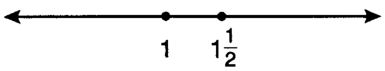
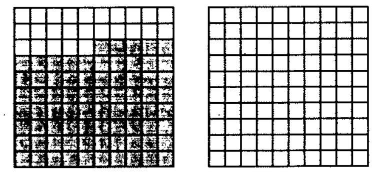
Problems that require students to think flexibly about rational numbers

a. If  is 3/4, draw the fraction strip for 1/2, for 2/3, for 4/3, and for 3/2. Be prepared to justify your answers.

b. 

Using the points you are given on the number line above, locate 1/2, 2 1/2, and 1/4. Be prepared to justify your answers.

c. 

Use the drawing above to justify in as many different ways as you can that 75% of the square is equal to 3/4 of the square. You may reposition the shaded squares if you wish.

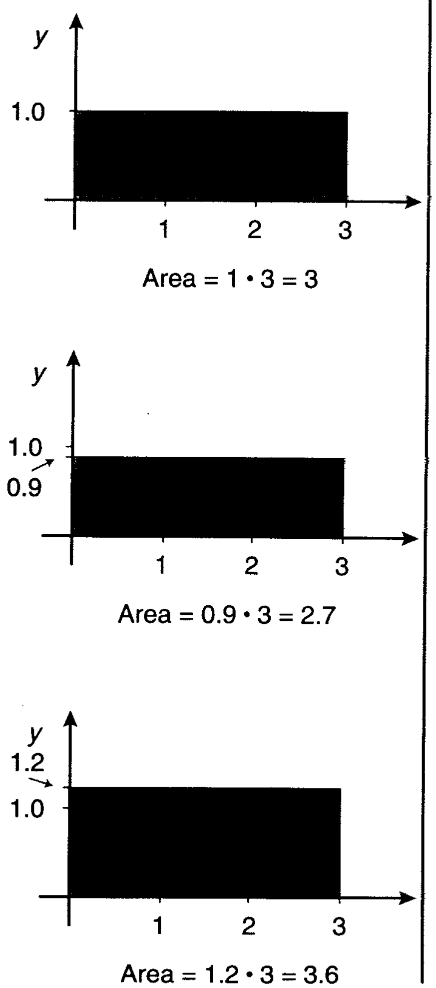
Only one rectangle can be made with seven tiles, so 7 is prime.

More than one rectangle can be made with eight tiles, so 8 is composite.

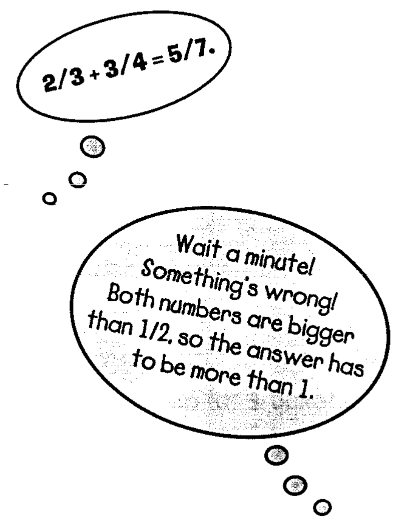
Tile explorations

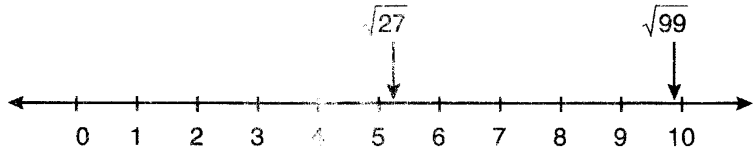
MATH 6-8
APPENDIX 3

This dynamic area model shows the effect of multiplying by a number less than 1.



Using benchmarks to estimate the results of a fraction computation



Locating square roots on a number line	 $\sqrt{27}$ is a little more than 5 because $5^2 = 25$. $\sqrt{99}$ is a little less than 10 because $10^2 = 100$.

Two approaches to solving a problem involving proportions

Which is the better buy—12 tickets for $15.00 or 20 tickets for $23.00?

Scaling Strategy

12 tickets for $15.00 → 60 tickets for $75.00.
20 tickets for $23.00 → 60 tickets for $69.00.

Unit-Rate Strategy

$15.00 for 12 tickets → $1.25 for 1 ticket.
$23.00 for 20 tickets → $1.15 for 1 ticket.

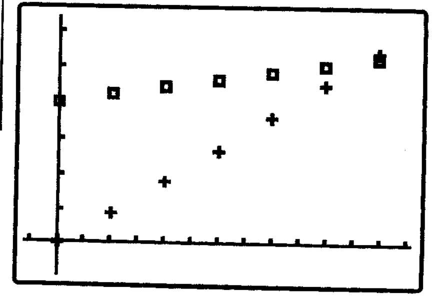
So 20 tickets for $23.00 is the better buy.

MATH 6-8
APPENDIX 6

Students can compare the charges for two telephone companies by making a table (a) and by representing the charges on a graphing calculator (b).

No. of minutes	0	10	20	30	40	50	60
Keep-in-Touch	$20.00	$21.00	$22.00	$23.00	$24.00	$25.00	$26.00
ChitChat	$0.00	$4.50	$9.00	$13.50	$18.00	$22.50	$27.00

(a)



(b)

MATH 6-8
APPENDIX I

A problem involving a nonlinear relationship, with an associated table and graph

Consider rectangles with a fixed area of 36 square units. The width (W) of the rectangles varies in relation to the length (L) according to the formula $W = 36/L$. Make a table showing the widths for all the possible whole-number lengths for these rectangles up to $L = 36$.

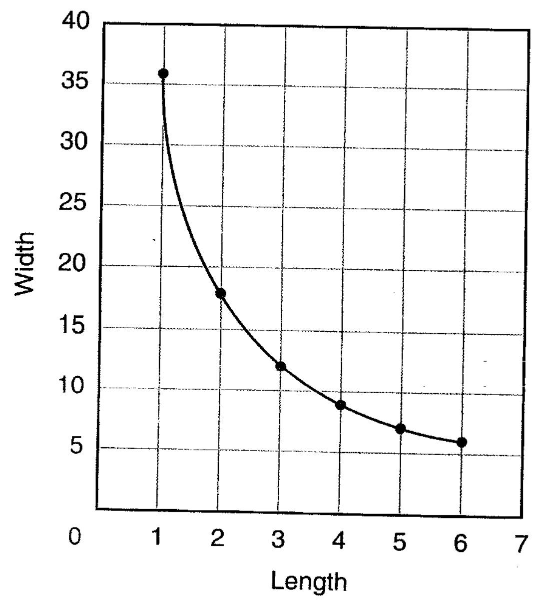
Solution:

Length	1	2	3	4	5	6	7	8	9	10	11	...	36
Width	36	18	12	9	7.2	6	5.14	4.5	4	3.6	3.27	...	1

Look at the table and examine the pattern of the difference between consecutive entries for the length and the width. As the length increases by 1, the width decreases, but not at a constant rate. What do you expect the graph of the relationship between L and W to look like? Will it be a straight line? Why or why not?

Solution:

The graph is not a straight line because the rate of change is not constant. Instead the graph appears to be a curve that bends sharply downward and then becomes more level.



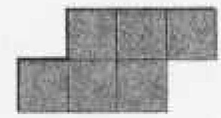


Fig. 9.1. Tile shapes

Working with square tiles, students can explore the question, "Can you add tiles to this figure [see fig. 9.1] to make a new figure with a perimeter of 18 units?" (Tiles must touch each other along an entire edge.)

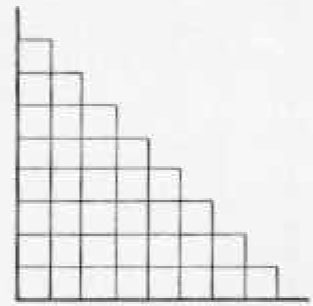


Fig. 9.2. Rectangles

Fig. 9.3. Width versus length

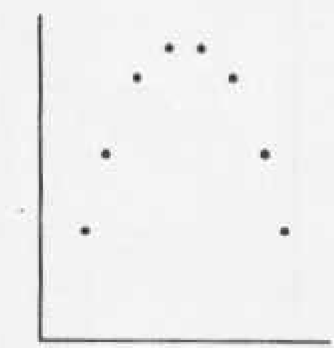
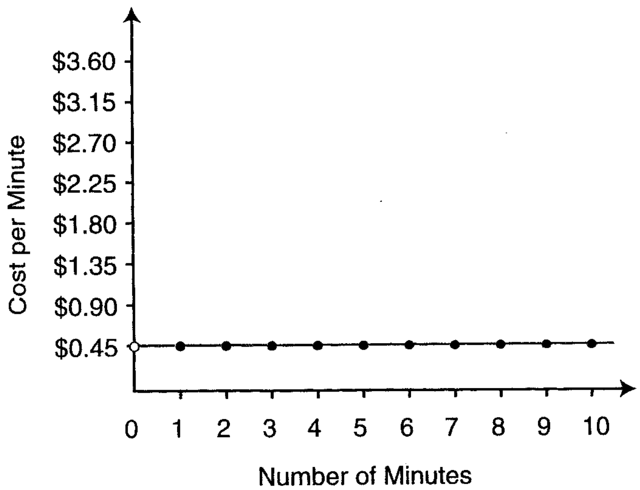
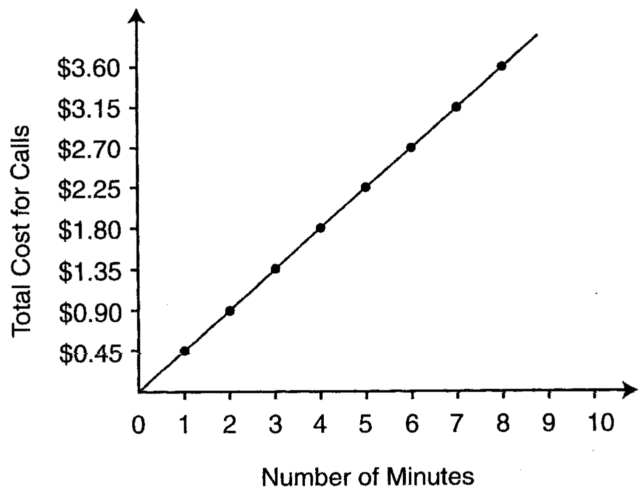


Fig. 9.4. Area versus length

These two graphs represent different relationships in ChitChat's pricing scheme.





MATH 6-8
APPENDIX 10

grades 5–8 incorrectly believe that if the sides of a figure are doubled to produce a similar figure, the area and volume also will be doubled. See figure 12.4.

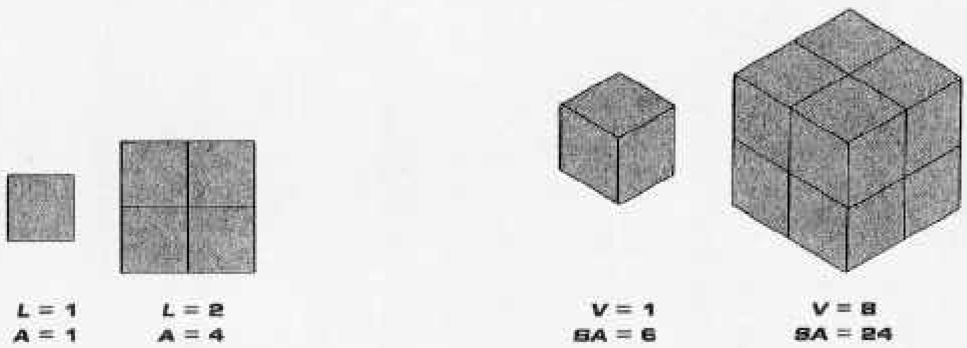
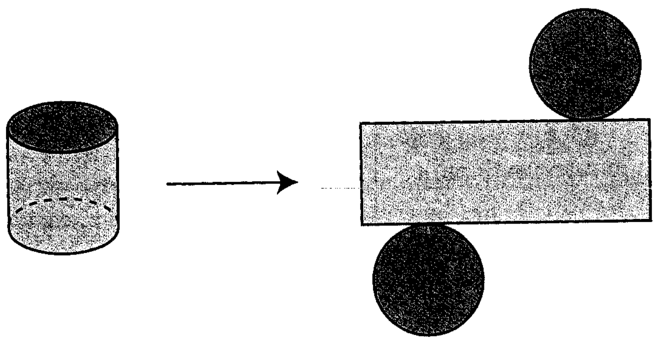


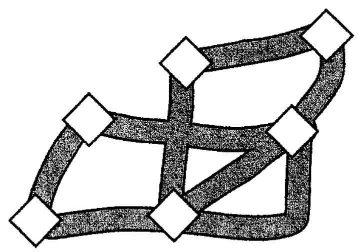
***Fig. 12.4.* Area and volume**

MATH 6-8
APPENDIX 11

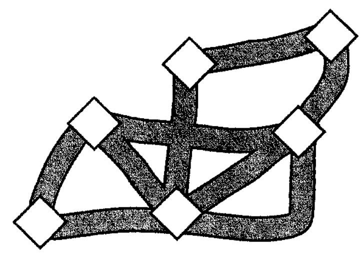


Students can determine the surface area of a cylinder by determining the area of its net.

Networks used to solve efficiency problems	Caroline's job is to collect money from parking meters. She wants to find an efficient route that starts and ends at the same place and travels on each street only once. A. The streets she has to cover are shown in map A. Find and trace such a route for her. B. A new street, shown in map B, may be added to her route. Can you find an efficient route that includes the new street?

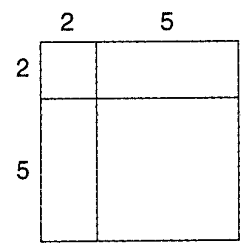


Map A



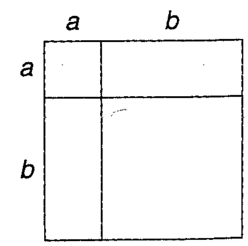
Map B

Geometric representation demonstrating the identity $(a + b)^2 = a^2 + 2ab + b^2$	(a) and (b) below



$$(2 + 5)^2 = 2 \cdot 2 + 2 \cdot 5 + 2 \cdot 5 + 5 \cdot 5$$
$$= 4 + 10 + 10 + 25$$
$$= 49$$

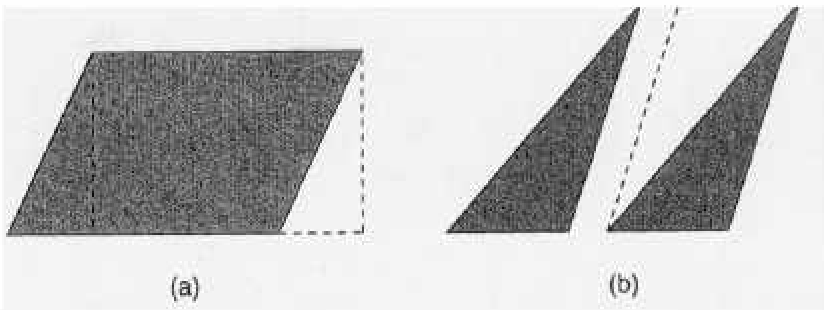
(a)

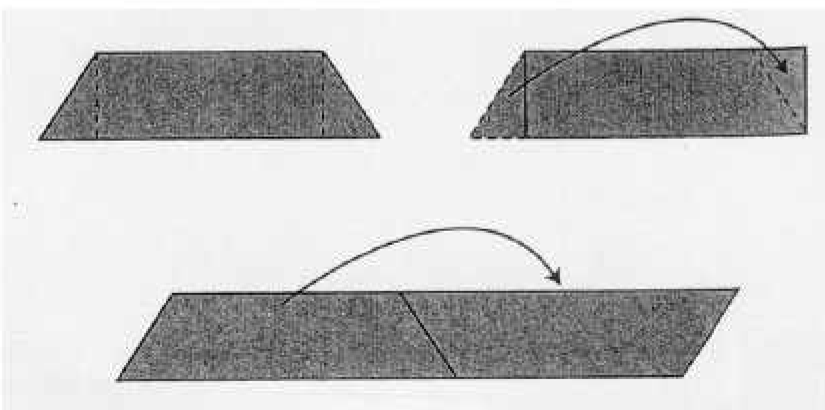


$$(a + b)^2 = a \cdot a + a \cdot b + a \cdot b + b \cdot b$$
$$= a^2 + 2ab + b^2$$

(b)

Students can use their knowledge of the area of a rectangle to generate a formula for the area of a parallelogram (a) and for the area of a triangle (b).

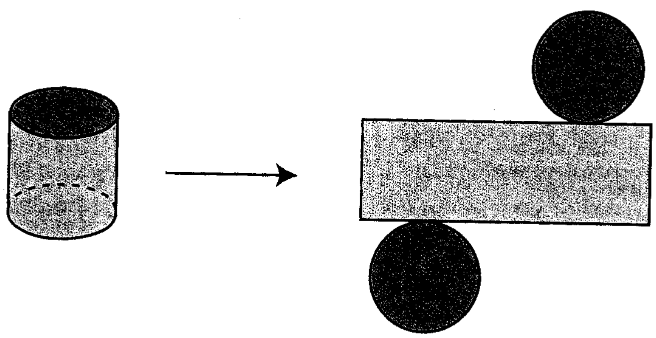




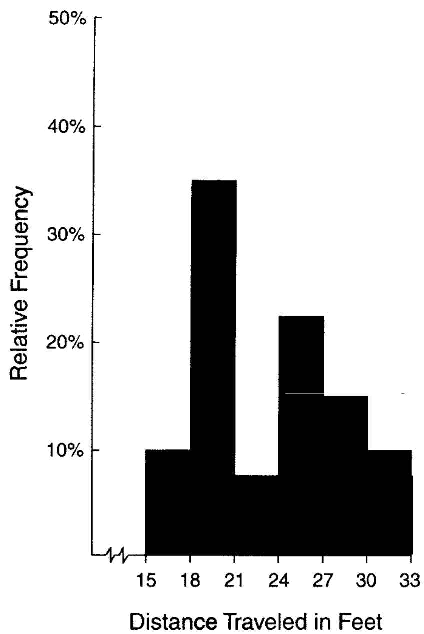
An isosceles trapezoid can be decomposed and rearranged or duplicated in order to find a formula for its area.

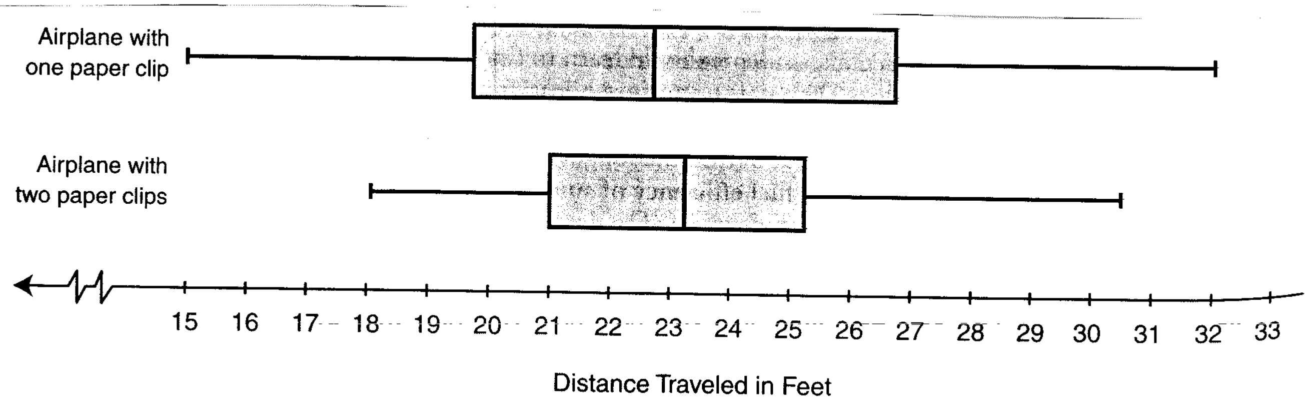
MATH 6-8
APPENDIX 15

Students can determine the surface area of a cylinder by determining the area of its net.



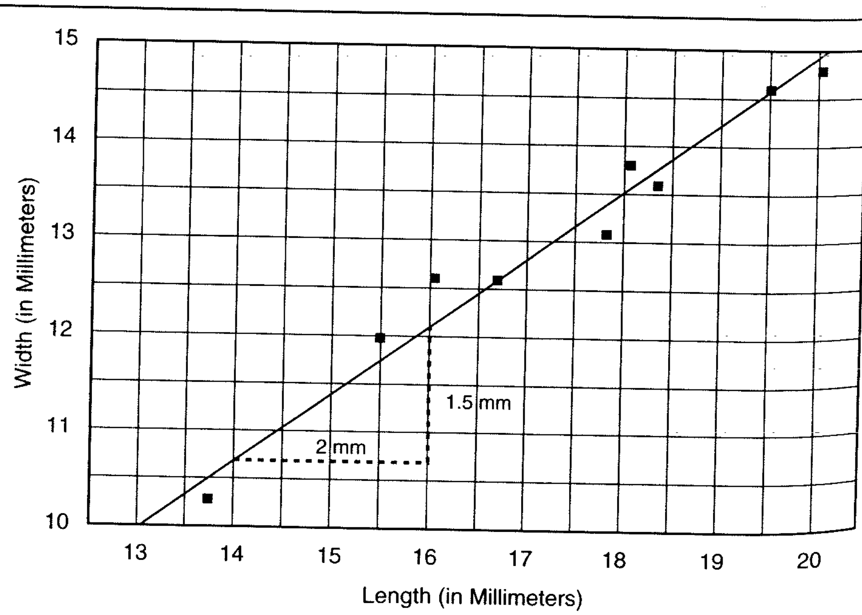
A relative-frequency histogram for data for a paper airplane with one paper clip





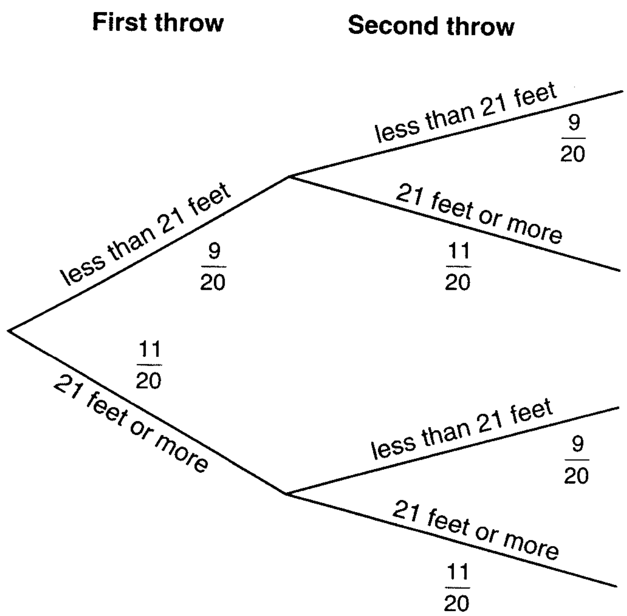
Math 6-8
Appendix 17

A scatterplot showing the relationship between the length and the width of warblers' eggs (Encyclopaedia Britannica Educational Corporation 1998, p. 109)



Math 6-8
Appendix 18

A tree diagram for determining the probability of a compound event, given sample data



$$\frac{9}{20} \times \frac{11}{20} = \frac{99}{400}$$

$$\frac{11}{20} \times \frac{9}{20} = \frac{99}{400}$$

$$\frac{99}{400} + \frac{99}{400} = \frac{198}{400} = .495$$